IV-UG-Math(CC)-VIII (NC)

2022

Full Marks - 60

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×8

   a) What is the multiplicity of 2 in $x^3 - 3x^2 + 4 = 0$

   b) What is the order of convergence of bisection method ?

   c) Define multiple root.

   d) What is the Lagrange's interpolation for $n = 2$?

   e) For constructing a polynomical of degree ≤ 5 how many nodes are needed ?

   f) What is the expression for nth divided difference in terms of fi ?

   g) What is the value of error constant in $S_{3/8}$ rule ?

   h) What is the value of f[a, b] in $f(x) = \frac{1}{x^3}$.

Part-II

2. Answer any ***eight*** of the following : $1\frac{1}{2} \times 8$

a) Find the interpolating polynominal corresponding to

x	0	1	2
y	1	0	−1

b) Define convergence of an interative method.

c) Construct the Newton's forward difference table for

x	1	2	3	4	5	6
f(x)	2	5	10	17	26	37

d) Consider the set of points (1, 2), (2, 5), (4, 7). By using linear spline find value of y at 3.

e) Find approximate value of $\int_0^1 x^2 dx$ using Trapezoidal rule taking step length $\frac{1}{3}$.

f) Find the polynomial of degree 2 that interpolates $y = x^3$ and the nodes $x_0 = 0, x_1 = 1, x_2 = 2$.

g) Write an iteration formula for finding $\sqrt{N}$, where N is a real number.

h) Define consistency of a system of linear system of algebraic equation $Ax = b$.

i) Determine the eigen values of

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

j) If $f(x) = \frac{1}{x^2}$, find the divided difference $f[x_1, x_2, x_3]$.

Part-III

3. Answer any *eight* of the following : 2 × 8

a) Find the 2nd approximate value of $x^4 - 5x + 1 = 0$ lying between 0 and 1 using Bisection method.

b) Derive the order of convergence of Newton Raphson method.

c) Consider set of points (1, 0), (2, 1) and (4, –1) by using quadratic spline interpolation find y(3) and y(1. 5).

d) Derive the error term for Trapezoidal rule.

e) Given $x_{n+1} = e^{-x_n}$, $x_0 = 0$. Do three steps using fixed point iteration method.

f) Using divided difference table construct quadratic polynomial of $f(x) = \sqrt{x}$ with nodes $x = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$.

g) Compute approximate values of y(1) for $y' = y(1 - y)$, $y(0) = \frac{1}{2}$ using Euler's method with $h = \frac{1}{4}$.

h) Find the smallest positive root of $x = e^{-x}$ correct of two decimal places using iterative method.

i) Construct the interpolating polynomial for interpolating $f(x) = x^3 + 2x^2 + 3x + 1$ at the nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$ and $x_5 = 5$.

j) With $h = \frac{1}{4}$ compute approximate value of $y(1)$ for $y' + 2y = 1, y(0) = 2$ using fourth order Runge Kutta method.

Part-IV

4. a) Obtain the positive real roots of $x^3 - 3x + 1 = 0$ correct to three decimal places using the method of false position. 6

OR

b) Find the smallest positive root of $x^2 - 5x + 1 = 0$ correct to three decimal places using fixed point iteration method.

5. a) Solve the system of equations

$$x_1 + 10x_2 - x_3 = 3$$
$$2x_1 + 3x_2 + 20x_3 = 7$$
$$10x_1 - x_2 + 2x_3 = 4$$

Using Gauss elimination with partial pivoting. 6

OR

b) Using Gauss Jordan method, find the inverse of

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$$

6. a) Given

x	14	17	31	35
f(x)	68.7	64.0	44.0	39.1

Find f(27) by using Lagrange interpolation formula. 6

OR

b) Using Newton's divided difference interpolation find y(10) given that y(5) = 12, y(6) = 13, y(9) = 14, y(11) = 16.

7. a) Using Simpson's $\frac{1}{3}$ rule, evaluate $I = \int_0^1 \frac{dx}{x^2 + 6x + 10}$ with 2 and 4 sub-intervals. Compare with the exact results. 6

OR

b) Using Trapzoidal rule evaluate $\int_0^{\pi} \sin x \, dx$ by dividing the range into 6 equal sub intervals.

2022

Full Marks - 80

Time - 3 hours

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Answer *all* questions

Part-I

1. Answer the following : 1×12

   a) Is $\phi \subset X$ open in the matric space (X, d) ?

   b) What is the interior of Q in R ?

   c) Let $D = \{(x, y) \in R^2 : x^2 + y^2 < 1\}$. Is D complete.

   d) If A, B are compact subsets of a metric space X. Is $A \cup B$ compact ?

   e) What is the interior of the subset $[0, 1] \subseteq R$.

   f) If $A = [0, 1]$ and $B = [1, 2]$ what is the value of $(A \cup B)^o$?

   g) By an example show that an arbitrary union of closed sets need not be closed.

h) Define fixed point of the mapping $T : X \to X$.

i) What is the point of discontinuity of [Sgn x] ?

j) Give an example of a function which is continuous only at one point.

k) Define diameter of a metric space.

l) Give an example of a compact set which is not bounded.

Part-II

2. Answer any ***eight*** of the following : 2×8

a) Show that $(C[0, 1], \langle\ ,\ \rangle)$ is an inner product space.

b) Show that any open interval (a, b) in R is an open ball.

c) Show that the map $x \to x^2$ from R to itself is continuous.

d) Show that any closed subset of a compact set in a metric space X is compact.

e) Show that the sequence $x_n = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$ is not Cauchy.

f) Let (X, d) be a metric space and A, B are subsets of X. Then show that $A \subseteq B \Rightarrow A^{o} \subseteq B^{o}$.

g) Show that the irrationals in R are of second category.

h) Discuss the continuing of $f(x) = \dfrac{1}{1 - e^{1/x}}$

i) Show that $f : (0, 1) \rightarrow R$ defined by $f(x) = 1/x$ is not uniformly continuous.

j) Show that the subset $A \subseteq R^2$ where $A = \{(x, y) : x^2 - y^2 \geq 4\}$ is disconnected.

Part-III

3. Answer any ***eight*** of the following : 3×8

a) Let (X, d) is a metric space. Define $\delta(x, y) = \min \{1, d(x, y)\}$ for all $x, y \in X$. Show that δ is a metric on X.

b) Show that :

$U = \{x, y) \in R^2 : xy \neq 0\}$ is open in R^2.

c) If $\{U_i : i \in I)$ be a family of open sets in (X, d). Show that $\bigcup_{i \in I} U_i$ is open in X.

d) Show that any constant map from a metric space to another is continuous.

e) Prove that $f : [1, \infty) \to R$ given by $f(x) = \frac{1}{x^n}, n \in N$ is uniformly continuous.

f) Show that

$$\left\{\frac{1}{n} : n \in N\right\} \cup \{0\}$$ is compact in R.

g) Show that the continuous image of a compact metric space is compact.

h) Let X be a nonempty set and $d(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$ show that (X, d) is a complete metric space.

i) Let (X, d) be a metric space and let F_1, F_2 be subsets of X. Then show that

$(F_1 \cup F_2)' = F_1' \cup F_2'$.

j) Let $f(x) = \sin {}^{1}\!/\!_{x}$, $x \in R \setminus \{0\}$. Show that $\lim_{x \to 0} f(x)$ does not exist. Hence show the function f can not be extended to a continuous function of R.

Part-IV

4. a) Prove that R is complete. 7

OR

b) Prove that any nonempty open set in R is the union of countable family of pairwise disjoint open intervals.

5. a) State and prove Baire's catagory theorem. 7

OR

b) Prove that every infinite set has a countable infinite subset.

6. a) Show that the closed ball $B = B[0, 1] \subset R^n$ is homeomorphic to the cube $Q = [-1, 1]^n \subset R^n$. 7

OR

b) Show that any uniformly continous function carries Cauchy sequence to Cauchy sequence. But the converse is not true.

7. a) Prove that any compact subset of a metric space is closed and bounded. 7

OR

b) Prove that any continuous function 'f' from a compact space (X, d) to another metric space (Y, d) is bounded i.e. f(X) is a bounded subset of Y.

2022

Full Marks - 80

Time - 3 hours

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Answer *all* questions

Part-I

1. Answer the following : 1×12

   a) What is the characteristic of the ring $z_2 \times z_4 \times z_6$?

   b) What is the cardinality of an finite integral domain ?

   c) Give an example of a commutative ring without unit element.

   d) Find idempotent elements of Z ?

   e) What are the maximal ideals of Z ?

   f) Is every prime ideal of R is a maximal ideal of R ?

   g) Define principal ideal domain.

   h) If f(x) and g(x) are two non zero polynomial then what is the value of deg f(x) g(x).

   i) List all the zero divisors of z_{13}.

   j) Find zeros of $x^2 + 3x + 2$ in z_6.

   k) Determine whether $x^4 + 3x + 3$ is irreducible over Q or not.

   l) How many elements are in $z[i] / \langle 3+i \rangle$?

Part-II

2. Answer any ***eight*** of the following : 2×8

   a) Show that $\{0\}$ is a prime ideal of Z.

   b) Find all factor rings of Q.

   c) State Einstein criterion for irreducibility.

   d) Let R be a ring then show that $(-a)b = a(-b) = -(ab)$ for all $a, b \in R$.

   e) Find all homomorphic images of Z.

   f) Show that $2x^3 + 6x^2 + 12x + 6$ is irreducible over Q.

   g) Show that Z/6Z is not a unique factorisation domain.

   h) Let R = {0, 2, 4, 6, 8} under addition and multiplication modulo 10. Write the addition table for R.

   i) Let a belongs to a ring R. Let $S = \{x \in R \mid ax = 0\}$. Show that S is a sub ring of R.

   j) Show that the function $f : Z_5 \rightarrow Z_{10}$ given by $f(x) = 3x$ is not a homomorphism.

Part-III

3. Answer any ***eight*** of the following : 3×8

   a) Show that the only ideals of a field F are $\{0\}$ and F itself.

   b) Le R be a ring with unity e. Then show that the mapping $\phi : z \rightarrow R$ given by $n \rightarrow ne$ is a ring homomorphism.

c) Show that the polynomial $x^{p-1} + x^{p-2} + \ldots + 1$ is irreducible over Q.

d) If a, b are associates in an integral domain D. Then prove that $\langle a \rangle = \langle b \rangle$ where $\langle a \rangle$ = ideal generated by a.

e) Prove that an element 'a' in a field F is a zero of $f(x) \in F[x]$ if and only if $(x - a)$ is a factor of $f(x)$ in $F[x]$.

f) Prove that cancellation law hold in a ring R if and only if R has no divisors of O.

g) Show that 13 is a reducible element in the ring z[i].

h) Find the quotient and remainder upon dividing $f(x) = 3x^4 + x^3 + 2x^2 + 1$ by $g(x) = x^2 + 4x + 2$.

i) If P is a prime number of the form $4n + 1$, solve the congruence $x^2 \equiv -1 \bmod P$.

j) Show that $Z[i] = \{m + ni : m, n \in Z\}$ where $i^2 = -1$ is an integral domain.

Part-IV

4. a) Prove that if a ring has unity, it is unique. If a ring has a multiplicative inverse, it is unique. 7

OR

b) Prove that the centre of a ring R is a subring of R.

5. a) Let $\phi : R \rightarrow S$ be a homomorphism of rings and let $I \subset R$ be its kernel. Then prove that $I_m \phi \cong R/I$. 7

OR

b) Let R be a commutative ring with unity and A be an ideal of R. Then prove that R/A is an integral domain if and only if A is prime.

6. a) In a principal ideal domain prove that an element is irreducible if and only if it is prime. 7

OR

b) Let F be a field. Then prove that f[x] is a principal ideal domain.

7. a) Define Eucledian domain. Prove that every Euceldian domain is a principal ideal domain. 7

OR

b) Let F be a field and let $P(x) \in F[x]$. Then prove that $\langle P(x) \rangle$ is a maximal ideal in F[x] if and only if P(x) is irreducible over F.

2022

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the folowing : 1×12

   a) What is the negative of the implication '$p \to q$' ?

   b) Suppose card $A = 1$. How many elements are in $P(P(A))$?

   c) State a relation which is symmetric, transitive but not reflexive on $A = \{1, 2, 3\}$.

   d) If $a = 58$, $b = 17$ express them as $a = bq + r$.

   e) What is gcd $(a, 0)$ where $a \pm 0$?

   f) What is the value of gcd . lcm of $(6, 21)$?

   g) Define skew symmetric matrix.

   h) What is the trivial solution for a homogeneous system $AX = 0$?

i) What is the inverse of $\begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix}$?

j) What are the conditions on a, b, c that (a, b, c) be in the null space of T ?

k) If eigen values of $A_{2\times 2}$ is λ_1 and λ_2 then what are the eigen values of A^{-1} ?

l) Define dimension of a vector space.

Part-II

2. Answer any *eight* of the following : 2×8

a) Determine whether "$-1 < x < 1 \leftrightarrow x^2 < 1$" is true or false

b) Construct a truth table for $p \rightarrow \sim(p \vee q)$.

c) For any sets A, B show that $(A \cup B)' = A' \cap B'$.

d) Show that 17369 and 5472 are relatively prime.

e) Given three consecutive integers a, a + 1, a + 2. Show that one of them is divisible by 3.

f) Determine whether $u = (3, 2, 1, 4)$, $v = (1, 2, 4, 5)$ are solutions of $x_1 + 2x_2 - 4x_3 + 3x_4 = 15$.

g) If $B = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{bmatrix}$ find det B^T ?

h) Show that $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basi for R^3 ?

i) Examine whether $A = \begin{bmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{bmatrix}$ is invertible or not ?

j) Find the characteristic polynomial of

$$A = \begin{bmatrix} 1 & 2 & 9 \\ 12 & 11 & 2 \\ 0 & 0 & 4 \end{bmatrix}.$$

Part-III

3. Answer any ***eight*** of the following : 3×8

a) Show that $[p \wedge q] \wedge [(\sim p) \vee (\sim q)]$ is a contradiction.

b) Let A and B are nonempty sets. Prove that $A \times B = B \times A$ if and only if $A = B$.

c) Define $f : z \to z$ by $f(x) = 3x^3 - x$. Show that f is one-to-one but not onto.

d) Show that any nonempty set of natural numbers has a smallest element.

e) Find q and r in the division algorithm $a = bq + r$ where $a = 98764$, $b = 4789$.

f) Write $v = (1, 3, 2)$ as a linear combination of $u_1 = (1, 2, 1)$, $u_2 = (2, 6, 5)$, $u_3 = (1, 7, 8)$.

g) Determine the cofactors of all elements of

$$A = \begin{bmatrix} 2 & -2 & 2 & 1 \\ -3 & 6 & 9 & -1 \\ 1 & -7 & 10 & 2 \end{bmatrix}.$$

h) Suppose A is a 2×1 matrix and B is a 1×2 matrix. Prove that $C = AB$ is not invertible.

i) Prove that if two vectors are linearly dependent, one of them is a scalar multiple of other.

j) Let T be a function from F^3 to F^3 defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$ Verify that T is a linear transformation.

Part-IV

4. a) Show that the statement

$$[p \vee [(\sim r) \rightarrow (\sim s)] \vee [(s \rightarrow [(\sim t) \vee p]) \vee ((\sim q) \rightarrow r)]$$

is neither a tautology nor a contradiction. 7

OR

b) For sets A and B prove that

i) $A \cap B = A$ if and only if $A \subseteq B$

ii) $A \cup B = B$ if and only if $A \subseteq B$.

5. a) Using mathematical induction establish the truth of $\frac{1}{1.5}+\frac{1}{5.9}+\frac{1}{9.13}+\ldots+\frac{1}{(4n-3)(4n+1)}$ $=\frac{n}{4n+1}$ for all $n \geq 1$. 7

OR

b) If $a = bq + r$ for integers a, b, q and r then prove that gcd (a, b) = gcd (b, r).

6. a) Row reduce $A=\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 3 \end{bmatrix}$ to echelon form. 7

OR

b) Solve $x + y + 3z = 1$

$$2x + 3y - z = 3$$

$$5x + 7y + z = 7$$

by row reduction method.

7. a) Prove that the subspace spanned by a nonempty subset 'S' of a vector space 'V' is the set of all linear combination of vectors in 'S'. 7

OR

b) If w_1 and w_2 are finite dimensional subspaces of a vector space V then prove that $w_1 + w_2$ is finite dimensional and

$$\dim w_1 + \dim w_2 = \dim (w_1 \cap w_2) + \dim (w_1 + w_2).$$

2019

Full Marks - 40

Time - 2 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

1. a) Construct the truth table for An(B Δ C). 6

b) i) Define Proposition with an example. 2

ii) Find the converse of the gollowing statement :

If the triangle is equiangular, then it is equilateral. 2

OR

c) Prove that the statement

$P \wedge (P \rightarrow q) \rightarrow q$ is a tautology. 6

d) i) Define Contrapositive of a statement 2

ii) Find the negation of the statement :

If triangle ABC is right triangle, then $|AB|^2 + |BC|^2 = |AC|^2$. 2

2. a) Prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$ 6

b) i) If Gopal is clever, then he is rich.

Find the coverse and inverse of the above statement. 2

ii) Write the negation :

It is raining and Mahanadi is flooded. 2

OR

c) Suppose that a class consist of a set S of 100 students, 70 of which pass in geometry and 60 pass in algebra. If no one failed in both the subjects, determine the number of students who passed in both algebra and geometry. 6

d) i) Define conjunction and disjunction of two statements. 2

ii) Draw the truth table for statement

$\sim q \rightarrow \sim p.$ 2

3. a) Prove that Congruence modulo relation on integers is an equivalence relation. 6

b) i) What do you mean by quantifier ? 2

ii) What is reflexive relation ? 2

OR

c) Show that

$$B-\bigcup_{n=1}^{n} A_i = \bigcap_{i=1}^{n} (B-A_i).$$ 6

d) i) Write the differences between finite and infinite sets. 2

ii) What is Predicate ? 2

4. a) How many integer between 10 and 100 (both inclusive) consist of distinct odd digits ? 6

b) i) Define equivalence relation. 2

ii) If $A = \{a, b\}$. Find the power set of A. 2

OR

c) Prove that the relation "greater than or equal to" on Z is a partial ordering relation. 6

d) i) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$

Let $f = \{(x, y)\} \in A \times B : x < y\}$

Find all elements of f. 2

ii) Define one-one relation and one-many relation with examples. 2